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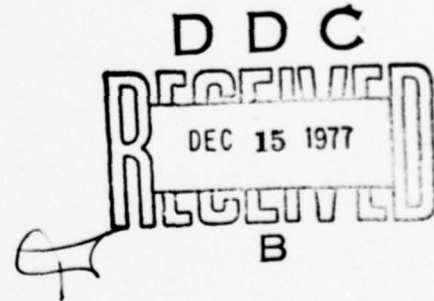
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SCATTERING OF SOUND BY INTERNAL WAVES: THE ROLE OF PARTICLE VELOCITIES

By: W. H. MUNK F. ZACHARIASEN

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ABSTRACT

Internal waves scatter sound by two related perturbations: (1) those associated with vertical particle displacements $\zeta(x,y,z,t)$ in the presence of a vertical gradient of (potential) sound speed: $\delta c = \zeta \cdot \partial_z c_p$; (2) those associated with horizontal particle velocities $u(x,y,z,t)$. The combined perturbations in refractive index are $\delta c/c + u/c$. The second term, which has been generally neglected, does become important under two rather special circumstances: (a) in sound transmission along a deep downward loop, and (b) in the *difference* between transmission from a source at point A to a receiver at point B and transmission from a source at B to a receiver at A.

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I. PROPAGATION VELOCITY

Let $\zeta(x,y,z,t)$ designate the upward particle displacement due to internal wave motion, and

$$u, v, w = \partial_t \zeta \quad (1)$$

the three components of internal wave particle velocities. The perturbation in sound velocity arising from the vertical displacement is

$$\delta c = \partial_{z_p} c_p \cdot \zeta \quad (2)$$

where $\partial_{z_p} c_p$ is the potential sound velocity gradient. Thus

$$\delta c \pm u \quad (3)$$

is the i.w. perturbation in *propagation velocity* in the $\pm x$ -direction, ignoring the small tilt of sound rays. The refraction parameter is

$$\mu^\pm = \frac{\delta c}{c} \pm \frac{u}{c} \quad (4)$$

For a canonical ocean,¹ Eq. (2) can be written

$$\frac{\delta c}{c} = 24.5 \text{ g}^{-1} n^2(z) \zeta \quad (5)$$

where $n = n_0 e^{z/b}$ (6)

is the Brunt-Väisälä (or buoyancy) frequency. Constancy of i. w. energy flux requires

$$\text{rms } \zeta \sim n^{-1/2}, \quad \text{rms } u \sim n^{1/2},$$

hence $\delta c/c \sim n^{3/2}$.

For orientation, with a sound axis at $z = -b = -1$ km, the following numerical values obtain:²

| | z | \tilde{n} | rm ζ | rms u | rms $\delta c/c$ | rms u/c |
|-------------|------|-------------|------------|--------|------------------|---------------|
| | km | cph | m | cm/sec | $\times 10^5$ | $\times 10^5$ |
| Thermocline | 0 | 3 | 7.3 | 4.70 | 49 | 3.1 |
| Sound axis | -1 | 1.1 | 12.0 | 2.85 | 11 | 1.9 |
| | -2 | .406 | 19.8 | 1.73 | 2.4 | 1.1 |
| bottom | -4.5 | .094 | 41.2 | 0.83 | 0.3 | 0.6 |

Thus δc and u contributions from i.w. are comparable beneath 2 km.

II. SPECTRA

We first refer to the previous results for the effects of vertical particle displacements. From MZ (115) and MZ (118) we have $\langle \phi^2 \rangle = \langle |X|^2 \rangle$. Then from (MZ66)

$$\langle \phi \phi \rangle = \frac{2}{\pi} q^2 n_o b \int_0^R dx \sec^2 \theta \sum_j j^{-1} \int_{\omega_L}^n d\omega (\omega^2 - \omega_L^2)^{-\frac{1}{2}} S(\omega, j; z) \quad (1)$$

$$\text{with} \quad \sum_j \int_{\omega_{in}}^n d\omega S(\omega, j) = \left\langle \left(\frac{\delta c}{c} \right)^2 \right\rangle \quad (2)$$

We now generalize (2.1) to

$$\langle \phi^+ \phi^+ \rangle = \dots \int \dots \sum \dots \int \dots S^{++}(\omega, j; z) \quad (3)$$

and similarly for $\langle \phi^- \phi^- \rangle$, $\langle \phi^+ \phi^- \rangle$, where

$$\sum_j \int_{\omega_{in}}^n d\omega \begin{bmatrix} S^{++}(\omega, j) \\ S^{--}(\omega, j) \\ S^{+-}(\omega, j) \end{bmatrix} = \begin{bmatrix} \langle \mu^+ \mu^+ \rangle = \left\langle \frac{\delta c}{c} \frac{\delta c}{c} \right\rangle + \left\langle \frac{u}{c} \frac{u}{c} \right\rangle + 2 \left\langle \frac{\delta c}{c} \frac{u}{c} \right\rangle \\ \langle \mu^- \mu^- \rangle = \langle \quad \rangle + \langle \quad \rangle - 2 \langle \quad \rangle \\ \langle \mu^+ \mu^- \rangle = \langle \quad \rangle - \langle \quad \rangle \end{bmatrix} \quad (4)$$

It is convenient to denote the contributions to the spectrum by

$$S\left(\frac{\delta c}{c} \frac{\delta c}{c}\right), \quad S\left(\frac{u}{c} \frac{u}{c}\right), \quad S\left(\frac{\delta c}{c} \frac{u}{c}\right) \quad (5)$$

so that

$$S^{++} \equiv S(\mu^+ \mu^+) = S\left(\frac{\delta c}{c} \frac{\delta c}{c}\right) + S\left(\frac{u}{c} \frac{u}{c}\right) + 2S\left(\frac{\delta c}{c} \frac{u}{c}\right),$$

and similarly for S^{--} and S^{+-} , using the sign convention in (2.4). In accordance with the Garrett and Munk internal wave model we write (in the notation of MZ)

$$\begin{aligned} S\left(\frac{\delta c}{c} \frac{\delta c}{c}\right) &= \left\langle \frac{\delta c}{c} \frac{\delta c}{c} \right\rangle G_{\zeta\zeta}(\omega) H(j) \\ S\left(\frac{u}{c} \frac{u}{c}\right) &= \left\langle \frac{u}{c} \frac{u}{c} \right\rangle G_{uu}(\omega) H(j) \\ S\left(\frac{\delta c}{c} \frac{u}{c}\right) &= \left\langle \frac{\delta c}{c} \frac{u}{c} \right\rangle G_{u\zeta}(\omega) H(j) \end{aligned} \quad (6)$$

$$\text{with } G_{\zeta\zeta} = \frac{4\omega_{in} (\omega^2 - \omega_{in}^2)^{1/2}}{\pi\omega^3}, \quad G_{uu} = \frac{4\omega_{in} (\omega^2 + \omega_{in}^2)}{3\pi\omega^3 (\omega^2 - \omega_{in}^2)^{1/2}} \quad (7)$$

$$G_{u\zeta} = 2\omega_{in}^2 \omega^{-3}, \quad H(j) = (j^2 + j_{**}^2)^{-1} / \sum_1^\infty (j^2 + j_{**}^2)^{-1}$$

$$\text{such that } \int_{\omega_{in}}^n d\omega G(\omega) = 1 \quad \sum_{j=1}^\infty H(j) = 1.$$

Further,

$$\left. \begin{aligned} \left\langle \frac{\delta c}{c} \frac{\delta c}{c} \right\rangle &= \left\langle \frac{\delta c}{c} \frac{\delta c}{c} \right\rangle_o \left(\frac{n}{n_o}\right)^3, & \left\langle \frac{\delta c}{c} \frac{\delta c}{c} \right\rangle_o &= \left(\frac{24.5 n_o^2}{g}\right)^2 \langle \zeta_o^2 \rangle = 24.0 \times 10^{-8} \\ \left\langle \frac{u}{c} \frac{u}{c} \right\rangle &= \left\langle \frac{u}{c} \frac{u}{c} \right\rangle_o \frac{n}{n_o}, & \left\langle \frac{u}{c} \frac{u}{c} \right\rangle_o &= \frac{3n_o^2}{2c^2} \langle \zeta_o^2 \rangle = 0.10 \times 10^{-8} \\ \left\langle \frac{\delta c}{c} \frac{u}{c} \right\rangle &= \left\langle \frac{\delta c}{c} \frac{u}{c} \right\rangle_o \left(\frac{n}{n_o}\right)^2, & \left\langle \frac{\delta c}{c} \frac{u}{c} \right\rangle_o &= I \frac{24.5 n_o^2}{g} \sqrt{\frac{3}{2}} \frac{n_o}{c} \langle \zeta_o^2 \rangle = 1.55 \times 10^{-8} \text{ I} \end{aligned} \right\} \quad (8)$$

A. The $u\zeta$ variance for internal waves

The function I is to allow for the phase relation between u and ζ . We shall demonstrate that for free internal waves u and ζ are in fact in quadrature, hence $I=0$. But it is to be expected that for forced (growing) internal waves this is not the case, and then $0 \leq I \leq 1$. In the latter case there would be much interest in *measuring* the $u\zeta$ and $u\dot{\zeta} = uw$ covariances, as a possible indication of the vertical transfer of horizontal momentum associated with internal waves.

Constant n . The simplest demonstration is for this case; the progressive wave solution gives³

$$\frac{u}{\omega a} = \sin\theta \cos\phi \cos(k_x x + k_z z - \omega t) + \frac{\omega_i n}{\omega} \sin\theta \sin\phi \sin(k_x x + k_z z - \omega t)$$

$$\zeta = a \cos\theta \sin(k_x x + k_z z - \omega t)$$

where $k_x = k_H \cos\phi$, $k_y = k_H \sin\phi$, $k_z = k_H \tan\theta$.

But there is evidence that the waves are very nearly standing waves (equal energy flux up and down); furthermore, McComas⁴ has demonstrated that if this is not the case there is strong nonlinear interaction to make it so. To obtain the standing wave solution we reverse the sign of both θ and k_z and add:

$$\begin{aligned}\frac{u}{\omega a} &= -2 \sin\theta \cos\phi \sin k_z z \sin(k_x x + \omega t) \\ &\quad + 2 \frac{\omega_{in}}{\omega} \sin\theta \sin\phi \sin k_z z \cos(k_x x - \omega t), \\ \zeta &= 2a \cos\theta \cos k_z z \sin(k_x x - \omega t).\end{aligned}$$

For progressive waves

$$\langle u\zeta \rangle = \frac{1}{2} a^2 \omega_{in} \sin\theta \cos\theta \sin\phi$$

and this vanishes for a horizontally isotropic field, as assumed GM75 $\frac{1}{2}$, since $\langle \sin\phi \rangle = 0$. For standing waves $\langle u\zeta \rangle$ is proportional to $\langle \sin\phi \rangle$ and $\langle \cos\phi \rangle$ and so again vanishes for a horizontally isotropic field. But in addition, u and ζ are in quadrature vertically, and this again leads to cancellation.

Suppose the upward traveling waves (downward energy) have an amplitude a , and the downward traveling waves ra . Then it follows that

$$I = (1-r) \langle \sin\phi \rangle$$

which vanishes in the case of vertical and horizontal isotropy.

General case. One might imagine that the foregoing result is a consequence of the assumption that n is constant, or of the WKB approximation in which n varies only slowly. It is, however, quite general. Let $\zeta(\vec{k}, \omega; z)$ be the Fourier component of vertical displacement corresponding to an internal wave with horizontal wavenumber \vec{k} and frequency ω . Thus

$$\zeta(\vec{k}, \omega; z) = \int d^2\vec{x} \int dt e^{-i(\vec{k}\vec{x} - \omega t)} \zeta(\vec{x}, t)$$

and (the complex quantity) $\zeta(\vec{k}, \omega; z)$ satisfies the equation

$$\left(\partial_z^2 + k^2 \frac{n^2(z) - \omega^2}{\omega^2 - \omega_{in}^2} \right) \zeta(\vec{k}, \omega; z) = 0 \quad (9)$$

for any buoyancy frequency $n(z)$. The boundary conditions normally used are that ζ should vanish at the surface and on the bottom of the ocean. The quantity of interest to us is $\langle u_z \rangle$. From the equations of motion, we have

$$\vec{u}(\vec{k}, \omega; z) = \left(\frac{\omega}{k^2} \vec{k} - i \frac{\omega_{in}}{k^2} \hat{e}_z x \vec{k} \right) \partial_z \zeta(\vec{k}, \omega; z)$$

where \hat{e}_z is a unit vector pointed vertically upward. It is then easy to show that

$$\begin{aligned} \langle u_z \rangle &= \frac{1}{2} \left\langle \frac{\omega}{k^2} \vec{k} \cdot \partial_z |\zeta(\vec{k}, \omega; z)|^2 \right\rangle \\ &+ \frac{1}{2} \left\langle -\frac{\omega_{in}}{k^2} \hat{e}_z x \vec{k} \cdot \text{Im}(\zeta^*(\vec{k}, \omega; z) \partial_z \zeta(\vec{k}, \omega; z)) \right\rangle. \end{aligned}$$

Now when averaged over a vertical distance small compared to the distance over which $\zeta(\vec{k}, \omega; z)$ varies appreciably, the first term is zero. And from the differential equation (9) we can readily derive the "conservation law" that

$$\text{Im}(\vec{\zeta}^*(\vec{k}, \omega; z) \partial_z \vec{\zeta}(\vec{k}, \omega; z))$$

is a constant independent of depth. Since in particular it vanishes on the bottom, it vanishes everywhere. Thus $\langle \vec{u} \vec{\zeta} \rangle$ is zero for any buoyancy frequency $n(z)$, without approximation, provided only that (9) holds.

3. MEAN-SQUARE PHASE

Substitution of (2.6) into (2.1) leads to

$$\langle \phi \phi \rangle = 2\pi^{-1} \frac{n_o}{\omega_{in}} q^2 b \langle j^{-1} \rangle \int_0^R dx \sec^2 \theta \left\langle \left(\frac{\delta c}{c} \right)^2 \right\rangle \int_{\omega_{in}}^n d\omega (\omega^2 - \omega_L^2)^{-1/2} \omega_{in} G_{\zeta \zeta}(\omega) \quad (1)$$

plus two other terms involving $\langle (u/c)^2 \rangle$ and G_{uu} , and $\langle (\delta c/c)(u/c) \rangle$ and $G_{u\zeta}$. Here $\langle j^{-1} \rangle = \Sigma j^{-1} H(j)$ is the i.w. weighted average of j^{-1} . The ω -integrations can be carried out explicitly.⁵ Writing

$$\omega_L^2 \equiv \omega_{in}^2 + n^2 \tan^2 \theta \equiv \omega_{in}^2 D^2, \quad y = \omega_{in}^2 / \omega^2$$

we have for the three ω -integrals

$$\left. \begin{aligned} \frac{1/D^2}{\pi} \int_{\omega_{in}^2/n^2}^{\omega_{in}^2} dy \sqrt{\frac{1-y}{1-Dy}} &= \frac{2}{\pi} \left(\frac{1}{D^2} + \frac{D^2-1}{2D^3} \log \frac{D+1}{D-1} \right) \\ \frac{2}{3\pi} \int dy \frac{1+y}{\sqrt{(1-y)(1-Dy)}} &= \frac{2}{3\pi} \left(-\frac{1}{D^2} + \frac{3D^2+1}{2D^3} \log \frac{D+1}{D-1} \right) \\ \int dy \sqrt{\frac{y}{1-Dy}} &= \frac{\pi}{2} \frac{1}{D^3} \end{aligned} \right\} \quad (2)$$

The x-integration in Eq. (3.1) has in general been done numerically by integration along the ray path. It can be done analytically in two special cases, as follows.

A. The axial approximation

Here $D=1$, $\theta=0$, $n=n_1$. We note that the second integral (involving $\langle uu \rangle$) becomes logarithmically infinite. This is treated in the Appendix. The result then is

$$\begin{aligned} \langle \phi^+ \phi^+ \rangle &= \frac{2}{\pi} \frac{n_0}{\omega_{in}} \langle j^{-1} \rangle q^2 b R \left[\frac{2}{\pi} \left\langle \left(\frac{\delta c}{c} \right)_1^2 \right\rangle + \right. \\ &\quad \left. + \frac{8}{3\pi} \log \frac{0.43 R_0}{R-R_0} \left\langle \left(\frac{u}{c} \right)_1^2 \right\rangle + 2 \frac{\pi}{2} \left\langle \left(\frac{\delta c}{c} \frac{u}{c} \right)_1 \right\rangle \right] \end{aligned} \quad (3)$$

where the subscript 1 denotes the sound axis value, and similarly for $\langle \phi^- \phi^- \rangle$ and $\langle \phi^+ \phi^- \rangle$. Here $R_0 = 21$ km is the range of an axial loop. The logarithmic singularity is weak, and unless we are within 100 m of an axial loop, the relative contributions of the three terms are determined by the averages $\langle ()_1 \rangle$. From Eq. (2.8), setting $n_1/n_0 = e^{-1}$, the relative magnitudes are

$$\begin{aligned} 24 \times 10^{-8} \cdot e^{-3} &= 1.2 \times 10^{-8} \\ 0.10 \times 10^{-8} \cdot e^{-1} &= .04 \times 10^{-8} \\ 4\pi^{-2} I \cdot 0.06 \times 10^{-8} &= .02 \times 10^{-8} I \end{aligned}$$

The first term, involving $\delta c/c$, clearly dominates.

B. The turning point approximation

Next we discuss the apex approximation. For steep rays we note that the major contributions to the ray integrals in $\langle \phi^+ \phi^+ \rangle$ come from the (upper and lower) apexes of the ray. Near an apex located at (\hat{x}, \hat{z}) , the equation of the ray is

$$z(x) = \hat{z} - (x - \hat{x})^2/2$$

$$\text{where } \kappa^{-1} = \gamma_A |1 - \exp(z - z_1)/b|, \quad \gamma_A = 1.14 \times 10^{-2} \text{ km}^{-1} \quad (\text{MZ85})$$

is the radius of curvature of the ray at the turning point. With this expression, we may approximately evaluate the ray integrals and obtain for the contribution of each apex

$$\langle \phi^+ \phi^+ \rangle_{\text{apex}} = 2q^2 b \gamma_A \langle j^{-1} \rangle \left\{ \left\langle \left(\frac{\delta c}{c} \right)_0^2 \right\rangle \left(\frac{\hat{n}}{n_0} \right)^2 + \left\langle \left(\frac{u}{c} \right)_0^2 \right\rangle + 2 \left\langle \left(\frac{\delta c}{c} \frac{u}{c} \right)_0 \right\rangle \left(\frac{\hat{n}}{n_0} \right) \right\} \quad (4)$$

where \hat{n} denotes the value of $n(z)$ at the apex: $\hat{n} = n(\hat{z})$. Thus the $(\delta c/c)_0^2$ and $(\delta c/c \cdot u/c)_0$ terms get their major contributions from upper turning points, while the $(u/c)_0^2$ has comparable contributions from both (in fact somewhat larger from the lower turning point since the radius of curvature κ is larger there).

C. Reciprocal transmissions

Thus in any one-way experiment other than for a ray with a single deep loop) the effect of particle velocities can be ignored. If, however,

we simultaneously transmit in opposite directions, then the three terms can be separately evaluated by constructing the three combinations

$$\begin{aligned} & \frac{1}{4} \left[\langle \phi^+ \phi^+ \rangle + \langle \phi^- \phi^- \rangle + 2\langle \phi^+ \phi^- \rangle \right] \\ & \frac{1}{4} \left[\langle \phi^+ \phi^+ \rangle + \langle \phi^- \phi^- \rangle - 2\langle \phi^+ \phi^- \rangle \right] \quad (5) \\ & \frac{1}{4} \left[\langle \phi^+ \phi^+ \rangle - \langle \phi^- \phi^- \rangle \right] , \end{aligned}$$

respectively. The implications of this are considered in Section IV.

D. Worcester's reciprocal experiment⁶

This corresponds to a lower turning point at 1.5 km, and a rather small scale-depth, $B \approx 0.75$ km. Thus $\hat{n}/n_0 = e^{-2}$, and the three terms in Eq. 3.4 are of a magnitude

$$\langle \phi^+ \phi^+ \rangle \sim \{0.44 + 0.10 + 0.42 I\} \times 10^{-8} .$$

Suppose that $I=0$, then we have

$$\langle \phi^+ \phi^+ \rangle \sim 0.54$$

$$\langle \phi^- \phi^- \rangle \sim 0.54$$

$$\langle \phi^+ \phi^- \rangle \sim 0.34$$

$$\langle (\phi^+ - \phi^-)^2 \rangle \sim 0.40$$

This is consistent with some preliminary results that $\langle \phi^+ \phi^+ \rangle$, $\langle \phi^- \phi^- \rangle$ and $\langle (\phi^+ - \phi^-)^2 \rangle$ have comparable magnitudes.

IV. RECIPROCAL COVARIANCES

An experiment involving reciprocal shooting will yield not only the mean-square quantities, but also the frequency content. To interpret such a spectrum we shall want to reverse the integration order, and do the x -integration first. Let

$$\langle \phi^+ \phi^+ \rangle = \int d\omega E^{++}(\omega)$$

etc. Then the last term of the Eq. (3.1) can be written

$$\begin{aligned} \frac{1}{4} \int d\omega [E^{++}(\omega) - E^{--}(\omega)] &= E(\omega) = \frac{4}{\pi} \langle j^{-1} \rangle q^2 B \bar{R} \frac{n^2}{\omega_{in} n_o} \left\langle \left(\frac{\delta c}{c} \frac{u}{c} \right) \right\rangle_o \\ &\cdot \frac{1}{R} \int_0^R dx \sec^2 \theta (\omega^2 - \omega_L^2)^{-1/2} G_{u\zeta}(\omega) \theta(n-\omega) \theta(\omega - \omega_L) \end{aligned}$$

where $\theta(x)$ is a step function, $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$. These θ functions restrict the part of the integral on the ray which can contribute to the spectrum for any given frequency. At any position x along the ray, the only frequencies which can interact with the ray (within the stationary phase approximation) are those satisfying

$$n(z(x)) > \omega > \sqrt{\omega_{in}^2 + n^2(z(x)) \tan^2 \theta(x)}$$

Thus when the ray is steep (θ is large) frequencies near ω_{in} are excluded. But when one is near the ray apex (n large and θ small) all frequencies between ω_{in} and $\hat{n} = n(\hat{z})$ are allowed.

One may, if one wishes, pick out different parts of the ray by forming the difference between the spectrum at two nearby frequencies. If $\Delta\omega \ll \omega$ is a small frequency difference, then for frequencies well above ω_{in} we have

$$\Delta E(\omega) \equiv E(\omega + \Delta\omega) - E(\omega) = -\frac{8}{\pi} \left\langle \left(\frac{\delta c}{c} \frac{u}{c} \right)_0 \right\rangle \left\langle \frac{1}{j} \right\rangle \frac{q^2 B \omega_{in} n^2}{n_o^4 \omega} \cdot \Delta\omega \left(1 / \frac{\partial n}{\partial x} \right)_{\omega=n} ;$$

thus $\Delta E(\omega)$ receives all of its contributions from that part of the ray where $n(z(x)) \approx \omega$.

V. MOMENTUM FLUX

We may wish to interpret the results of reciprocal transmission without relying so heavily on internal waves. To this end let us express the measured quantities in terms of the spectrum of momentum flux in the ocean without making any assumptions regarding the origin of this spectrum.

Let $\hat{}$ designate averages along a ray. Thus⁷

$$\hat{u}(t) = R^{-1} \int ds \, u(s,t)$$

$$\hat{\zeta}(t) = R^{-1} \int ds \, \zeta(s,t)$$

We may regard \hat{u} and $\hat{\zeta}$ as the measured quantities for they are so closely related to the measured travel times; let

$$T^{\pm} = - (R/\bar{c}) \, \hat{u}^{\pm} = - (R/\bar{c}^2) [\delta\hat{c} \pm \hat{u}] \quad ,$$

designate the departures from the mean travel time (averaged over several inertial periods). Then

$$\delta\hat{c} = \partial_{z_p} c \cdot \hat{\zeta} = - (\bar{c}^2/R) \, \frac{1}{2}(T^+ + T^-)$$

$$\hat{u} = - (\bar{c}^2/R) \, \frac{1}{2}(T^+ - T^-)$$

(Actually the vertical gradient of potential sound velocity should be properly depth weighted.)

Consider the time-lagged covariance

$$\hat{C}_{u\zeta}(\tau) = \langle \hat{u}(t) \hat{\zeta}(t-\tau) \rangle$$

Can we manipulate \hat{C} to get some measure of the momentum flux $\langle uw \rangle$?

First we note that

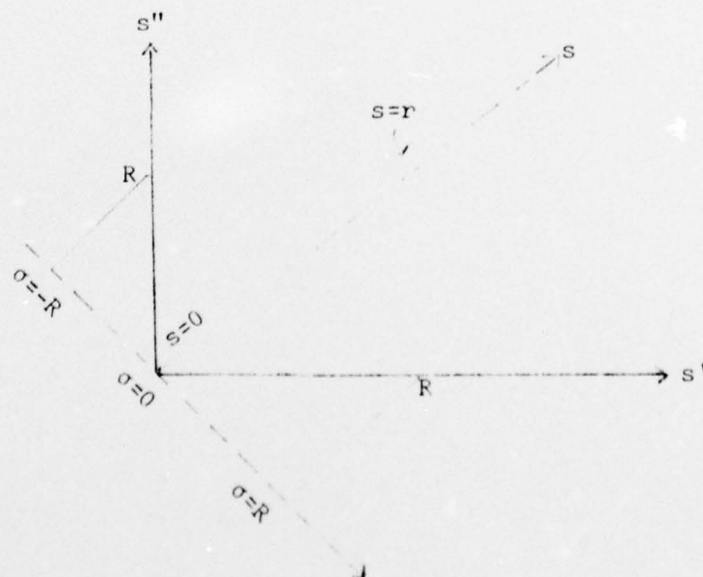
$$\begin{aligned} \partial_{\tau} \hat{C}_{u\zeta}(\tau) &= \langle \hat{u}(t) \partial_{\tau} \hat{\zeta}(t-\tau) \rangle \\ &= - \langle \hat{u}(t) \partial_t \hat{\zeta}(t-\tau) \rangle = -\hat{C}_{uw}(\tau) \end{aligned}$$

Now

$$\hat{C}_{u\zeta}(\tau) = R^{-2} \int ds' \int ds'' C_{u\zeta}(s, s'; \tau)$$

where

$$C_{u\zeta}(s', s''; \tau) = \langle u(s', t) \zeta(s'', t-\tau) \rangle$$



At this point let us simplify the geometry to horizontal rays. Then switching to center of mass and relative coordinates

$$s = \frac{1}{2}(s' - s''), \quad \sigma = s' - s''$$

We have

$$\begin{aligned} \int_0^R ds' \int_0^R ds'' &= \int_0^R d\sigma \int_{\sigma/2}^{R-\sigma/2} ds + \int_{-R}^0 d\sigma \int_{\sigma/2}^{R+\sigma/2} ds \\ &= \int_0^R d\sigma (R - \sigma) + \int_{-R}^0 d\sigma (R + \sigma) \end{aligned}$$

The second line follows if the integrand is a function only of σ , not of s . Under these conditions

$$C_{u\zeta}(\sigma, \tau) = \langle u(s, t) \zeta(s - \sigma, t - \tau) \rangle$$

$$\begin{aligned} \hat{C}_{u\zeta}(\tau) &= R^{-2} \int_{-R}^R d\sigma (R - |\sigma|) C_{u\zeta}(\sigma, \tau) \\ \hat{C}_{uw}(0) &= R^{-2} \int_{-R}^R d\sigma (R - |\sigma|) C_{uw}(\sigma, 0) \\ &= -2R^{-2} \int_0^R d\sigma \sigma C_{uw}(\sigma, 0) \end{aligned}$$

Flux spectrum. The more general question is to interpret the covariance of the "measured" quantities:

Let
$$C_{\hat{u}\hat{\zeta}}(\tau) = \langle \hat{u}(t) \hat{\zeta}(t-\tau) \rangle$$

$$F_{\hat{u}\hat{\zeta}}(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} C_{\hat{u}\hat{\zeta}}(\tau) \quad .$$

Further

$$C_{u\zeta}(\sigma, \tau) = \langle u(s, t) \zeta(s-\sigma, t-\tau) \rangle$$

$$= \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i(k\sigma - \omega\tau)} F_{u\zeta}(k, \omega)$$

so that

$$F_{u\zeta}(k, \omega) = \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\tau e^{-i(k\sigma - \omega\tau)} C_{u\zeta}(\sigma, \tau) \quad .$$

Hence

$$\begin{aligned}
 F_{u\zeta}^{\wedge}(\omega) &= R^{-2} \int_0^R ds' \int_0^R ds'' \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} e^{i(k\sigma - \omega\tau)} F_{u\zeta}(k, \omega') \\
 &= R^{-2} \int_0^R ds' \int_0^R ds'' \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik\sigma} F_{u\zeta}(k, \omega) \\
 &= R^{-2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} F_{u\zeta}(k, \omega) \left[\int_{-R}^0 d\sigma (R+\sigma) e^{ik\sigma} + \int_0^R d\sigma (R-\sigma) e^{-ik\sigma} \right] \\
 &= R^{-2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} F_{u\zeta}(k, \omega) 2 \int_0^R d\sigma (R-\sigma) \cos k\sigma \\
 &= \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{2}{k^2 R^2} (1 - \cos kR) F_{u\zeta}(k, \omega)
 \end{aligned}$$

The Reynolds spectrum $F_{uw}(k, \omega) = i\omega F_{u\zeta}(k, \omega)$, and so will be related to the quadrature spectrum of the measured quantities.

REFERENCES

- ¹W. Munk, "Sound Channel in an Exponentially Stratified Ocean; With Application to SOFAR," J. Acoust. Soc. Am. **55**, 220-226 (1974).
- ²W. H. Munk and F. Zachariassen, "Sound Propagation Through a Fluctuating Stratified Ocean: Theory and Observation," J. Acoust. Soc. Am. **59**, 818-838 (1976), to be referred to as MZ. Table 1 of MZ is in error with regard to u/c .
- ³For example, O. M. Phillips, *The Dynamics of the Upper Ocean* (Cambridge University Press, 1966), Eq. 5.4.6.
- ⁴C. H. McComas III, "Equilibrium Mechanisms Within the Oceanic Internal Wave Field," J. Phys. Oceanogr. (in press).
- ⁵I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, 1965). Formula 2.264 is helpful. For the first integral set $x = 1-y$ and multiply numerator and denominator by \sqrt{x} .
- ⁶P. F. Worcester, "Reciprocal Transmission in a Mid-ocean Environment," J. Acoust. Soc. Am. (in press).
- ⁷For the time being we are sloppy about distinguishing between projections and arc lengths *along* the tilted ray.
- ⁸W. Munk, J. Acoust. Soc. Am. (1974) Eq. 24.

APPENDIX

The second integral in (3.2) diverges as $D \rightarrow 1$. Set

$$\Delta^2 = D^2 - 1 = (n/\omega_{in})^2 \tan^2 \theta .$$

Then the integral approaches

$$I = \frac{8}{3\pi} \log \frac{2}{\Delta} . \quad (1)$$

We can express Δ as a function of range. Let source and receiver be on the axis at $x = \pm \frac{1}{2} R$. Then with

$$\delta z = A \cos \pi x/R$$

designating the distance of the ray above the sound axis, we find⁸

$$A = - \frac{3\pi}{4} B \frac{R-R_0}{R_0}$$

where $R_0 = \frac{1}{2} \pi B \epsilon^{-1/2} = 21 \text{ km}$ refer to the axial loop ($R < R_0$ for upward loops). It follows that

$$\theta = \frac{\delta z}{\delta x} = \frac{3\pi^2}{4} \frac{B}{R} \frac{R-R_0}{R_0} \sin \frac{\pi x}{R}$$

$$\Delta = \frac{n_1 \theta}{\omega_{in}} = \frac{3\pi^2}{4} \frac{n_1}{\omega_{in}} \frac{B}{R} \frac{R-R_0}{R_0} \sin \frac{\pi x}{R} .$$

Setting $B = 1 \text{ km}$, $n_1 = 1.9 \times 10^{-3} \text{ sec}^{-1}$, $\omega_{in} = 7.3 \times 10^{-5} \text{ sec}^{-1}$,
 $R_0 = 21 \text{ km}$, we have

$$\Delta = 9.2 \frac{R-R_0}{R_0} \sin \frac{\pi x}{R} .$$

We now require

$$\begin{aligned} \frac{1}{R} \int_{-\frac{1}{2}R}^{\frac{1}{2}R} dx \, I &= \frac{8}{3\pi R} \int_{-\frac{1}{2}R}^{\frac{1}{2}R} dx \left[\log \frac{2R_0}{9.2(R-R_0)} + \log 2 \right] \\ &= \frac{8}{3\pi} \log \left[\frac{4R_0}{9.2(R-R_0)} \right] . \end{aligned}$$

Thus even if we are within 0.1 km of the convergence distance, $R-R_0 = 0.1 \text{ km}$, the effect of currents is still relatively small. So the logarithmic singularity is of no practical interest.

As an aside, the singularity arises because $(\omega^2 - \omega_L^2)^{-\frac{1}{2}} \rightarrow (\omega^2 - \omega_{in}^2)^{-\frac{1}{2}}$ and $G_{uu} \sim (\omega^2 - \omega_{in}^2)^{-\frac{1}{2}}$, so that the integrand of $I_{uu} \sim (\omega^2 - \omega_{in}^2)^{-1}$, and I_{uu} is logarithmically infinity. This depends then on the fact that GM-spectrum for u has a $(\omega^2 - \omega_{in}^2)^{-\frac{1}{2}}$ cusp (but, of course, the integral of the spectrum and hence $\langle u^2 \rangle$ are finite). If the GM-spectrum had a $(\omega^2 - \omega_{in}^2)^{-s}$ cusp, with $s < \frac{1}{2}$, then the logarithmic infinity for I'_{uu} does not arise. The evidence for the $-\frac{1}{2}$ exponent is very weak. Observational evidence clearly shows a cusp (see for example Carl Wunsch, 1975, *JGR* **80**, 339-343, Fig. 1) and for an integrable cusp "... we require $0 < s < 1$. We arbitrarily choose the mid-point $s = \frac{1}{2}$." (GM72, p. 249). But

subsequently Cairns and Williams (Part II, Fig. 5) find the vertical displacement spectrum (which is a less critical test than a u-spectrum) in better accord with $s = 0.5$ than $s = 0.1$ and $s = 0.9$. The conclusion is that the singularity is probably too weak to be observable; otherwise, it would suggest an acceptable modification of the GM-spectrum.

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| Dr. Frederick Tappert Courant Institute 251 Mercer Street New York, NY 10012 | 1 | Dr. Kenneth Watson Lawrence Berkeley Laboratory University of California Berkeley, California 94720 | 1 |
| Capt. Peter R. Tatro, USN Office of the Oceanographer of the Navy 200 Stovall Street Alexandria, Virginia | 1 | Mr. J. Weileman Courant Institute 251 Mercer Street New York, NY 10012 | 1 |
| Dr. Alex Thompson Physical Dynamics, Inc. Berkeley, California | 1 | Dr. H. Weinberg Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320 | 1 |
| Mr. Richard D. Trueblood Electronics Engineer Commander Naval Ocean Systems Center San Diego, CA 92152 | 1 | Dr. M.S. Weinstein Underwater Systems, Inc. 8121 Georgia Avenue, Ste. 700 Silver Spring, MD 20910 | 1 |
| Dr. H. Uberall Catholic University 620 Michigan Avenue, NE Washington, D.C. 20017 | 1 | Maj. Gen. J.A. Welch, Jr. AFSA 1E 388 The Pentagon Washington, D.C. 20330 | 1 |
| University of Texas Applied Research Laboratory Austin, Texas 78712 | 2 | Capt. J.B. Wheeler Naval Electronic System Command Code 103 Washington, D.C. 20360 | 1 |
| Mr. R.J. Urick TRACOR, Inc. 1601 Research Blvd. Rockville, MD 20850 | 1 | LTC. William Whitaker Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, VA 22209 | 1 |

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| Mr. Peter Worcester University of California/San Diego E-25 La Jolla, California 92093 | 1 | | |
| Dr. J.L. Worzel Marine Science Institute Geophysics Laboratory 700 Thestrand Galveston, Texas 77550 | 1 | | |
| Dr. Carl Wunsch Massachusetts Institute of Technology Room 54-1324 Cambridge, Massachusetts 02139 | 1 | | |
| Dr. H. Yura Aerospace Corporation P.O. Box 92956 Los Angeles, CA 90009 | 1 | | |
| Dr. Frederik Zachariasen 452-48 Department of Physics California Institute of Technology Pasadena, California 91109 | 1 | | |